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Multi-jet cross sections in deep inelastic scattering at next-to-leading order

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Abstract

We present the perturbative prediction for three-jet production cross section in DIS at the NLO accuracy. We study the dependence on the renormalization and factorization scales of exclusive three-jet cross section. The perturbative prediction for the three-jet differential distribution as a function of the momentum transfer is compared to the corresponding data obtained by the H1 collaboration at HERA.

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We present the perturbative prediction for three-jet production cross section in DIS at the NLO accuracy. We study the dependence on the renormalization and factorization scales of exclusive three-jet cross section. The perturbative prediction for the three-jet differential distribution as a function of the momentum transfer is compared to the corresponding data obtained by the H1 collaboration at HERA.

Deep inelastic lepton-hadron scattering (DIS) has played a decisive role in our understanding of the deep structure of matter. The latest version of the experiment performed with colliding 27.5 GeV electrons or positrons and 820 GeV protons at HERA yields increasingly precise data so that not only fully inclusive measurements can be used to study the physics of hadronic final states. In fact, the study of multi-jet events and event shapes has become an important project at HERA [1].

One of the important theoretical tools in the analysis of hadronic final states is perturbative Quantum Chromodynamics (QCD). In order to make quantitative predictions in perturbative QCD, it is essential to perform the computations (at least) at the next-to-leading order (NLO) accuracy. In the case of DIS such computations have so far been completed for one-jet inclusive and 2(+1 beam)-jet cross sections [2]. In this letter we compute the 3+1-jet exclusive cross section, defined using the $k_\perp$ algorithm [3], as a function of the jet resolution parameter at the NLO accuracy. We also compute various differential distributions of the three-jet cross section, defined using the inclusive $k_\perp$ algorithm [4], that can be compared to recent data obtained by the H1 collaboration [5]. With our Monte Carlo program one can compute the NLO cross section of any other infrared safe two- and three-jet quantity — the particular distributions shown here are given simply as illustration.

An immediate application of our computation is, of course, the analysis of HERA multi-jet data. Using our program, it is also possible to find the higher order QCD correction to the forward jet cross section in the low-x regime in DIS, to which the $\mathrm{O}(\alpha_s^2)$ computation effectively gives only the leading-order prediction and large corrections may come from higher orders [6]. Our computation is also part of the $\mathrm{O}(\alpha_s^3)$ two-jet cross section.

In computing the NLO corrections we use the dipole formalism of Catani and Seymour [7] that we modify slightly in order to have a better control on the numerical computation. The main idea is to cut the phase space of the dipole subtraction terms as introduced in Ref. [8], the details of applying it to the case of DIS will be given elsewhere.

The advantages of using the dipole method are the followings: i) no approximation is made; ii) the exact phase space factorization allows full control over the efficient generation of the phase space; iii) neither the use of color ordered subamplitudes, nor symmetrization, nor partial fractioning of the matrix elements is required; iv) Lorentz invariance is maintained, therefore, the switch between various frames (e.g. laboratory and Breit frames) can be achieved by simply transforming the momenta; v) the use of crossing functions is avoided; vi) it can be implemented in an actual program in a fully process independent way.

In order to ensure the correctness of our results we checked the following points: (i) the correctness of all matrix elements and phase-space generation; (ii) the subtraction term regularizes the real correction in all possible soft and collinear limits; (iii) the same quantity is subtracted from the real correction as added to the virtual one.

We achieved full control on the matrix elements by utilizing the well-tested matrix elements that we had used in our program `DEBRECEN` [9] for computing NLO corrections to three- and four-jet production in electron-positron annihilation [8], with different particles crossed into the initial state. In particular, we used the crossing symmetric tree-level five-parton helicity amplitudes of Ref. [8] and the crossing symmetric three- and four-parton tree and one-loop amplitudes of Bern et al [10]. We consider only virtual photon exchange, in which case the necessary one-loop four-parton plus a gauge boson matrix elements were also derived by Glover et al [11]. Both sets of matrix elements were tested in $e^+e^- \to$ four jets NLO cross section computations and full agreement among the theoretical descriptions [12] and very good description of experimental data [13] were found.

We have checked numerically that in all soft and collinear regions the difference of the real and subtraction terms contain only integrable square-root singularities. Furthermore, we have also checked that our results are independent of the parameter that controls the volume of the cut dipole phase space, which ensures that indeed the same quantity has been subtracted from the real correction as added to the virtual one.

Finally, to have a further check of the computation, we utilized that the dipole method allows for the construction of a process independent programming of QCD jet cross sections at the NLO accuracy. We use the same program structure, with trivial modifications, to compute dijet and three-jet cross sections. Consequently, using the correct matrix elements and checking that our dijet cross sections are correct, we obtain the correctness of the three-jet result automatically.

In order to check the strucure of our program we compared our NLO two-jet predictions to those of two other existing programs [14,15]. In Ref. [15] a comparison of the these programs was presented using the NLO two-jet results for the modified JADE clustering scheme. In Table I. we recall the numbers presented there for one particular set of parton distribution functions together with the corresponding result of our computation. (We refer to Ref. [15] for the precise meaning of these numbers.) We find complete agreement with the predictions of `DISASTER++`. Our results also agree with the predictions of `DISENT` with slight tendency to lower values, but within statistical errors, from `DISENT` for small values of x_B and the inelasticity y (first two bins). A more complete comparison will be presented elsewhere.

TABLE I. Comparison of the NLO two-jet cross sections defined using the modified JADE algorithm and for MRSD$'_-$ parton densities obtained with three partonic Monte Carlo programs `DISENT`, `DISASTER++` and `NLOJET++` (this work).

bin	`DISENT`	`DISATER++`	`NLOJET++`
1	578.4 ± 7.1	585.0 ± 2.6	585.5 ± 2.0
2	361.1 ± 3.5	364.8 ± 1.5	364.8 ± 1.5
3	120.1 ± 0.9	119.1 ± 1.7	122.9 ± 0.5
4	95.4 ± 0.87	98.1 ± 1.11	97.7 ± 0.6
5	54.9 ± 0.40	55.3 ± 0.46	55.7 ± 0.4
6	17.3 ± 0.13	17.5 ± 0.06	17.5 ± 0.1
7	12.3 ± 0.15	12.1 ± 0.50	12.4 ± 0.07
8	8.52 ± 0.08	8.61 ± 0.12	8.61 ± 0.07
9	2.63 ± 0.02	2.65 ± 0.03	2.63 ± 0.02

In the case of DIS the observables may be defined either in the laboratory frame, in the hadronic center of mass frame (virtual boson plus proton rest frame), or in the Breit frame, which is characterized by purely space-like virtual boson momentum. In order to easily apply detector cuts, our program generates events in the laboratory frame. Due to the Lorentz invariance of the dipole method, we can freely transform the momenta to the other frames to compute the jet functions. Jet algorithms can be defined in any of these frames. The $k_\perp$ scheme we employ here is implemented also in the Breit frame [3]. The full algorithm is a two-step procedure, the first one is a preclustering of hadrons characterized by a pre-chosen stopping parameter $d_{\rm cut}$. For clustering the partons, we used the covariant E-scheme. The result of the first step is the beam jet plus hard final-state jets. The second step is the resolution of the hard final-state jets into sub-jets characterized by a parameter $y_{\rm cut}$. We scaled the stopping parameter with the momentum transfer squared Q^2 as $d_{\rm cut} = f_{\rm cut} Q^2$, and we chose $y_{\rm cut} = 1$, when the second step is not carried out. Thus our three-jet cross section is a function of one parameter, $f_{\rm cut}$.

Once the phase-space integrations are carried out, we write the three-jet cross section (at a given $f_{\rm cut}$) at NLO accuracy in the following form:

$$\sigma_{\rm 3jet}(f_{\rm cut}) = \sum_a \int {\rm d}\eta\, f_a(\eta,\mu_F) \Big[\overline{\alpha}_s(\mu_R)^2 B_a(\eta, f_{\rm cut}) + \overline{\alpha}_s(\mu_R)^3 \Big(B_a^{(R)}(\eta, f_{\rm cut}, \mu_R) + B_a^{(F)}(\eta, f_{\rm cut}, \mu_F) + C_a(\eta, f_{\rm cut})\Big)\Big] \,. \qquad (1)$$

In Eq. (1) $\overline{\alpha}_s = \alpha_s/2\pi$, $f_a(\eta,\mu_F)$ is the parton distribution function for parton type a at momentum fraction η and factorization scale μ_F. B_a gives the Born contribution, $B_a^{(R)}$, $B_a^{(F)}$, C_a are the correction functions. In the higher order correction we separated the dependence on the renormalization scale $\mu_R = x_R Q_{\rm H.S.}$ and factorization scale $\mu_F = x_F Q_{\rm H.S.}$, where $Q_{\rm H.S.}$ is the hard scattering scale. $Q_{\rm H.S.}$ is usually set event by event, therefore, the scale dependences can only be written in the factorized form if we consider differential cross section in $Q_{\rm H.S.}$. The $B_a^{(R)}(\eta, f_{\rm cut}, \mu_F)$ function is obtained by multiplying the Born squared matrix element with $\beta_0 \ln x_R^2$, where $\beta_0 = \frac{1}{3}(11 C_A - 2N_f)$, and integrating over the phase space. Apart from trivial factors the $B_a^{(F)}(\eta, f_{\rm cut}, \mu_F)$ function is obtained by convoluting the Altarelli-Parisi splitting functions with the integral of the colour-correlated Born squared matrix elements (see Eqs. (8.39) and (8.41) in Ref. [7]). The $C_a(\eta, f_{\rm cut})$ correction function requires much more computer time to compute.

Eq. (1) shows that using the dipole method one may either compute the full cross section at the NLO accuracy including the convolution with the parton distribution functions, or simply the parton level functions $B_a(\eta, f_{\rm cut})$, $B_a^{(F)}(\eta, f_{\rm cut})$ and $C_a(\eta, f_{\rm cut})$, which can then be convoluted with the parton densities after the Monte Carlo integration. The latter procedure is the proper one if we are interested in an α_s measurement from DIS data (to avoid the reculculation of the Monte Carlo integrals after each step of the fitting iteration). However, it requires at least triple differential binning (one for $f_{\rm cut}$, one for the momentum fraction η and one for the hard scattering scale $Q_{\rm H.S.}$) for the nine partonic functions (three for each of the three incoming flavor type), which is beyond the scope of a letter.

When presenting numerical results of three-jet cross sections in this paper we use the kinematic region used by the H1 collaboration [5]. For the basic DIS kinematic variables Q^2, $x_{\rm B}$ and $y = Q^2/(s\, x_{\rm B})$ we require

$$5\,\mathrm{GeV}^2 < Q^2 < 5000\,\mathrm{GeV}^2\,,$$
$$0 < x_{\mathrm{Bj}} < 1\,, \quad 0.2 < y < 0.6\,. \tag{2}$$

Furthermore, we restrict the (pseudo)rapidity-range in the laboratory frame and the minimum transverse energy of the jets in the Breit frame as

$$-1 < \eta_{\mathrm{jet,lab}} < 2.5\,, \qquad E_T > 5\,\mathrm{GeV}\,. \tag{3}$$

We choose the average transverse momentum of the jets,

$$Q_{\mathrm{H.S.}} = \frac{1}{3}\sum_j E_T^B(j) \tag{4}$$

as hard scattering scale. We also studied the other usual choice, when $Q^2_{\mathrm{H.S.}}$ is the negative of the momentum transfer carried by the virtual boson (Q^2), but have not found significant differences.

In Fig. 1, we plotted the cross section convoluted with the CTEQ5M1 parton distribution functions [16] and using the two-loop formula for the strong coupling,

$$\overline{\alpha}_s(\mu) = \frac{\overline{\alpha}_s(M_Z)}{w(\mu, M_Z)}\left(1 - \overline{\alpha}_s(M_Z)\,\frac{\beta_1}{\beta_0}\frac{\ln(w(\mu, M_Z))}{w(\mu, M_Z)}\right)\,, \tag{5}$$

where $w(q, q_0) = 1 - \beta_0\,\overline{\alpha}_s(q_0)\ln(q_0/q)$, $\overline{\alpha}_s(M_Z) = 0.118/2\pi$ and $\beta_1 = \frac{1}{3}(17C_A^2 - 6C_F T_R N_f - 10 C_A T_R N_f)$, with $N_f = 5$ flavors. For the leading order results we used the CTEQ5L distributions and the one-loop α_s ($\overline{\alpha}_s(M_Z) = 0.127/2\pi$ and $\beta_1 = 0$ in Eq. (5)). We used running $\overline{\mathrm{MS}}$ electromagnetic coupling at the scale of the virtual photon momentum squared. In order to show the weak dependence on the parton distribution functions, we plotted the perturbative predictions also using the MRST99 parton distribution functions [17] (dashed lines). There is no LO fit in the MRST99 set, therefore, in this latter case the LO curve is missing.

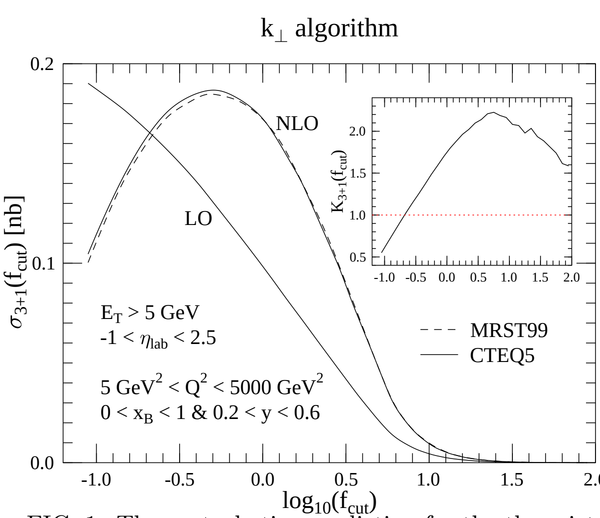

FIG. 1. The perturbative prediction for the three-jet cross section $\sigma_{3\mathrm{jet}}(f_{\mathrm{cut}})$ as a function of f_{cut}. The solid lines were obtained using the CTEQ5 parton distribution functions and the dashed lines with the MRST99 set.

The inset in Fig. 1 shows the K factor (ratio of the three-jet cross section at the NLO to that at the LO accuracy), indicating the relative size of the correction. The K factor exhibits a clear maximum, where the correction is more than 100 % and becomes smaller than one, implying negative correction, for $f_{\mathrm{cut}} < 0.2$. The position and height of the maximum as well as that of the point $K = 1$ depends on the lower limit on the Q^2 range. For instance, increasing the lower limit from $5\,\mathrm{GeV}^2$ to $100\,\mathrm{GeV}^2$, the point $K = 1$ moves to about $f_{\mathrm{cut}} = 0.03$. Negative NLO correction indicates that the resummation of large logarithms is necessary in order to obtain a reliable prediction for smaller values of the stopping parameter.

In Fig. 2 we study the scale dependences of the three-jet cross section at a fixed value of the resolution parameter $f_{\mathrm{cut}} = 0.2$. The strong dependence on the renormalization scale observed at LO is significantly reduced. The factorization scale dependence is already not significant at LO and does not change much. Setting the two scales equal, $\mu_R = \mu_F = \mu$, we can observe a wide plato peaking near $\mu = Q_{\mathrm{H.S.}}$.

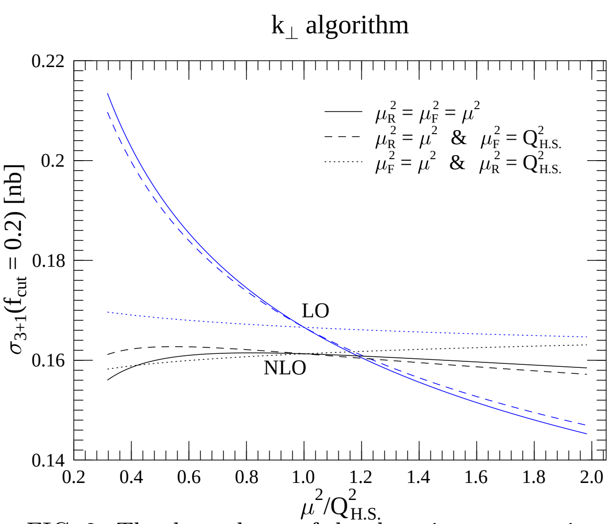

FIG. 2. The dependence of the three-jet cross section $\sigma_{3\mathrm{jet}}$ at the fixed value of $f_{\mathrm{cut}} = 0.2$ on the renormalization and factorization scales.

The first measurement of the three-jet cross sections in DIS was published by the H1 collaboration in Ref. [5]. They used the inclusive $k_\perp$ algorithm to define the jets (the precise definition is given in Ref. [18]), selected three-jet events and plotted differential distributions of certain kinematical variables. We computed the same distributions at the LO and NLO accuracy and compared the predictions, corrected to hadron level (the correction factors are about 0.8), to the published data. In Fig. 3 we present the $\mathrm{d}\sigma_{3\mathrm{jet}}/\mathrm{d}Q^2$ differential distribution. We observe that the LO prediction has a different shape than the data: too low for small values of Q^2 and too high at high values of Q^2. The radiative corrections bring theory and experiment much closer: the NLO prediction gives

a remarkably good description of the corrected experimental data. Similar conclusions can be drawn from the distributions of the other kinematical variables studied in Ref. [5], but not shown here.

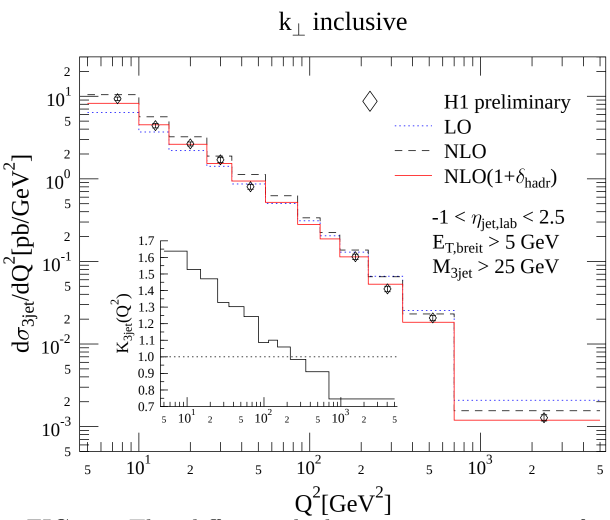

FIG. 3. The differential three-jet cross section for the $5 < Q^2 < 5000\,\mathrm{GeV}^2$ region compared to experimental data measured by the H1 collaboration. The LO prediction (dashed line) was obtained with the CTEQ5L parton distribution functions and the NLO prediction (solid line) was obtained with the CTEQ5M1 parton distribution functions.

In this letter we presented a NLO computation of the three-jet rate defined with the $k_\perp$ clustering algorithm in DIS. Our results were obtained using a partonic Monte Carlo program that is suitable for implementing any detector cuts. We found that the K factor is very sensitive to the allowed kinematic region. We demonstrated that the NLO corrections reduce the scale dependence significantly. The NLO prediction is weakly dependent on the parton distribution functions. The NLO perturbative prediction gives a remarkably good description of data measured at HERA and corrected to parton level. The same program can be used for computing the QCD radiative corrections to the (differential) cross section of any kind of two-, or three-jet cross section or event-shape distribution in DIS. We compared the two-jet rates obtained by our program to previous results and found agreement.

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